*Solution by the PROPOSER.

The variation of $\int Vdx$ may be divided into $\triangle u$ and $\triangle u_1$, the former arising supposing v constant, the latter from the variation of v. Thus $\triangle u = H + \int K \triangle y dx$, $\triangle u_1 = \int \frac{dV}{dv} \triangle v dx$. If the letters represent for V what their primes represent for V', $\triangle u = \int (N' \triangle y + P' \triangle p + \text{..........}) dx$. If $L = \frac{dV}{dv}$ and $I = \int L dx$,

$$\triangle u_1 = \int L \triangle v dx = I dv - \int I \frac{d\triangle v}{dx} dx = I(H' + \int K' \triangle y dx) - (Hi' + \int Ki' \triangle y dx)$$

(Hi, Ki denoting H' and K' when IN', IP', etc., are substituted for N', P',.....
Then

$$\triangle u + \triangle u_1 = \triangle y \left(P - \frac{dQ}{dx} \text{.....}\right) + \triangle p \left(Q - \frac{dR}{dx} + \text{.....}\right) + \text{..........}$$

$$+ \int \left(N - \frac{dP}{dx} + \frac{d^2 Q}{dx^2} - \text{.....}\right) \triangle y dx + I \triangle y \left(P' - \frac{dQ'}{dx} + \frac{d^2 R'}{dx^2} \text{.....}\right)$$

$$+ I \triangle p \left(Q' - \frac{dR'}{dx} + \text{.....}\right) + \text{.....} + I \int \left(N' - \frac{dP'}{dx} + \frac{d^2 Q'}{dx^2} \text{.....}\right) dx$$

$$- \triangle y \left(IP' - \frac{dIQ'}{dx} + \text{.....}\right) - \triangle p \left(IP' - \frac{dIQ'}{dx} + \text{.....}\right) - \text{..........}$$

$$- \int \left(IN' - \frac{dIP'}{dx} + \frac{d^2 IQ'}{dx^2} \text{.....}\right) dx.$$

Also solved by G. B. M. Zerr.

205. Proposed by Z. T. JACKSON, St. Louis, Mo.

Evaluate† $\int_0^{\frac{1}{2}\pi} \log \sin x \, dx$.

Solution by J. E. SANDERS, Reinersville, Ohio.

$$u = \int_0^{\frac{1}{2}\pi} \log \sin x \, dx = \int_0^{\frac{1}{2}\pi} \log \sin(\tfrac{1}{2}\pi - x) dx = \int_0^{\frac{1}{2}\pi} \log \cos x \, dx.$$

$$\therefore 2u = \int_0^{\frac{1}{2}\pi} (\log \sin x + \log \cos x) dx = \int_0^{\frac{1}{2}\pi} \log(\sin x \cos x) dx$$

$$= \int_0^{\frac{1}{2}\pi} \log \frac{\sin 2x}{2} dx = \int_0^{\frac{1}{2}\pi} \log \sin 2x \, dx - \frac{\pi}{2} \log 2.$$

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*See Williamson's *Integral Calculus*, Sixth Edition, p. 275.
†Byerly, *Integral Calculus*, p. 102.

Let $2x=x'$, then

$$\int_0^{\frac{1}{2}\pi}\log\sin 2x\,dx=\tfrac{1}{2}\int_0^{\pi}\log\sin x'\,dx'=\int_0^{\frac{1}{2}\pi}\log\sin x\,dx.$$

$$\therefore 2u=u-\frac{\pi}{2}\log 2,\quad u=-\frac{\pi}{2}\log 2=\frac{\pi}{2}\log\tfrac{1}{2}.$$

Also solved by M. E. Graber, G. W. Greenwood, L. E. Newcomb, and G. B. M. Zerr.

206. Proposed by DR. O. E. GLENN, Drury College.

Evaluate $\int_0^1(1-z^n)^m\frac{\partial}{\partial z}\log(1-z^nx^n)dz$, assuming $-1<x^n<+1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons. W. Va.

$u=-\int_0^1\frac{nx^nz^{n-1}(1-z^n)^m}{1-x^nz^n}dz$. Let $1-z^n=y$, then we have

$$u=-\int_0^1\frac{x^ny^m}{1-x^n+x^ny}dy=-\int_0^1\frac{y^m}{y+a}dy,\text{ where }\frac{1-x^n}{x^n}=a.$$

$$\therefore u=-\int_0^1\left(y^{m-1}-ay^{m-2}+a^2y^{m-3}\ldots\ldots(-1)^{m-1}a^{m-1}+\frac{(-1)^ma^m}{y+a}\right)dy$$

$$=-\left.\frac{y^m}{m}-\frac{ay^{m-1}}{m-1}+\ldots\ldots+(-1)^{m-1}a^{m-1}y+(-1)^ma^m\log(y+a)\right]_0^1$$

$$=-\left[\frac{1}{m}-\frac{a}{m-1}+\ldots\ldots+(-1)^{m-1}a^{m-1}+(-1)^ma^m\log\left(\frac{1+a}{a}\right)\right]$$

$$=-\left[\frac{1}{m}-\frac{(1-x^n)}{x^n(m-1)}+\frac{(1-x^n)^2}{x^{2n}(m-2)}+\ldots\ldots\frac{(-1)^{m-1}(1-x^n)^{m-1}}{x^{(m-1)n}}\right.$$

$$\left.+(-1)^{m+1}\frac{(1-x^n)^m}{x^{mn}}\log(1-x^n)\right].$$

DIOPHANTINE ANALYSIS.

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128. Proposed by F. P. MATZ, Ph. D., Sc. D., Reading, Pa.

Required the highest powers of 2, 3, 5, 7, contained in (1000)!

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

$$(1000)!=2^{500}(500)!(1.3.5\ldots\ldots999)$$
$$(500)!=2^{250}(250)!(1.3.5\ldots\ldots499)$$

....

Proceeding thus we find the powers required are

$$2^{994},\quad 3^{498},\quad 5^{249},\quad 7^{164}.$$